Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Engineering Mathematics-II

Subject Code: 4TE02EMT1 **Branch: B.Tech(All)**

Semester: II **Date: 19/11/2015**

Time: 10:30 To 1:30 **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) A square matrix A is called orthogonal if

(a) $AA^{-1} = I$ (b) $A^2 = A$ (c) $A^T = A^{-1}$ (d) $A^2 = I$

b) A $n \times n$ Non-Homogeneous system of equations $AX = B$ is given. If $\rho(A) = \rho(A:B) = n$ then the system has

(a) No solutions (b) Unique solutions
(c) Infinite solution (d) None of these

c) The rank of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 3 (d) -2

d) The Sum of the eigenvalues of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is

(a) 1 (b) 4 (c) 2 (d) 5

e) Find the value of $\begin{vmatrix} 1 & 2 & 3 \\ 0 & -2 & 3 \\ 1 & 0 & 0 \end{vmatrix} =$ _______

(a) 1 (b) 12 (c) -2 (d) 0

f) A square matrix A is called Singular if

(a) $|A| = 0$ (b) $A^2 = A$ (c) $AA^T = I$ (d) $|A| \neq 0$

g) $\int_{-\pi/2}^{\pi/2} \sin^7 x \; dx =$ _______

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{1}{2}$

h) $\int_0^{\pi/2} \cos^4 x \ dx =$ ________

(a) 0 (b) 1 (c) $\frac{3\pi}{16}$ (d) $\frac{8\pi}{3}$

i) $\int_0^1 \int_0^x dy \ dx =$ ________

(a) $\frac{1}{2}$ (b) -1 (c) 0 (d) y

j) The value of $\int_{-\pi}^{\pi} \sin mx \sin nx \ dx$ for $m \neq \pm n$ is

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π

k) Angle between the vectors $2i+2j-k$ and $6i-3j+2k$ is

(a) $\cos^{-1}\left(\frac{4}{11}\right)$ (b) $\cos^{-1}\left(\frac{4}{21}\right)$ (c) $\sin^{-1}\left(\frac{4}{11}\right)$ (d) $\cos^{-1}\left(\frac{4}{21}\right)$

l) div curl $\vec{V} =$ __________

(a) 0 (b) 1 (c) $\vec{0}$ (d) $\vec{V}$

m) A vector $\vec{F}$ is said to be irrotational if

(a) $\nabla \times \vec{F} = 0$ (b) $\nabla \cdot \vec{F} = 0$ (c) $\nabla \vec{F} = 0$ (d) None of these

n) If $\begin{bmatrix} x & 2 \\ 3 & 1 \end{bmatrix}$ is a singular matrix then $x =$ _______

(a) 1 (b) 6 (c) 2 (d) -6

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}$ by using determinant method. **(05)**

b) Evaluate : $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$ **(05)**

c) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$ to the normal form and find its rank. **(04)**

Q-3 Attempt all questions

a) Find the eigenvalues & eigenvectors of a matrix $A=\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ **(05)**

b) Solve the following system of equations by Cramer's rule: **(05)**

$$x+2y-z=3;\quad x+y+2z=9;\quad 2x+y-z=2$$

c) Determine $\int_0^1 \ln x\,dx$ converge or diverges. **(04)**

Q-4 Attempt all questions

a) Find the volume common to the cylinder $x^2+y^2=a^2$ and $x^2+z^2=a^2$. **(05)**

b) Find the inverse of the following matrix by using elementary transformation **(05)**

$$A=\begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 2 & 1 \\ 3 & -2 & 1 & 6 \end{bmatrix}$$

c) Solve: $\dfrac{dy}{dx}+y\tan x=\sin 2x,\ \ y(0)=1$ **(04)**

Q-5 Attempt all questions

a) Obtain Row echelon & Reduced row echelon form of the following matrix: **(05)**

$$A=\begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

b) Solve: $\dfrac{dy}{dx}=2y\tan x+y^2\tan^2 x$ **(05)**

c) Find the directional derivatives of $\phi=xy^2+yz^2$ at the point $(2,-1,1)$ in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$. **(04)**

Q-6 Attempt all questions

a) Evaluate $\int_C \bar{F}\,d\bar{r}$ where $\bar{F}=(x^2+y^2)\hat{i}-2xy\hat{j}$ and C is the rectangle in the xy-plane bounded by $y=0, x=a, y=b, x=0$. **(05)**

b) Evaluate $\iint_S \bar{F}\cdot\hat{n}\,ds$, where $\bar{F}=18z\hat{i}-12\hat{j}+3y\hat{k}$ and S is the part of the plane $2x+3y+6z=12$ in the first octant. **(05)**

c) Solve the system of equation by Gauss-Elimination method. **(04)**

$$2x+2y+2z=0$$
$$-2x+5y+2z=1$$
$$8x+y+4z=-1$$

Q-7 Attempt all questions

a) Change the order of integration and evaluate $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx\,dy$. **(05)**

b) Solve: $\left(x+\dfrac{ay}{x^2+y^2}\right)dx+\left(y-\dfrac{ax}{x^2+y^2}\right)dy=0$ **(05)**

c) Evaluate: $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}\left(x^2+y^2+z^2\right)dz\,dy\,dx$ **(04)**

Q-8 Attempt all questions

a) Verify Green's theorem for $\oint_C\left[(x-y)\,dx+3xy\,dy\right]$ where C is the boundary of the region bounded by the parabolas $x^2=4y$ and $y^2=4x$. **(07)**

b) State Cayley-Hamilton theorem and Find the characteristic equation for the matrix $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$. Also find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$. **(07)**